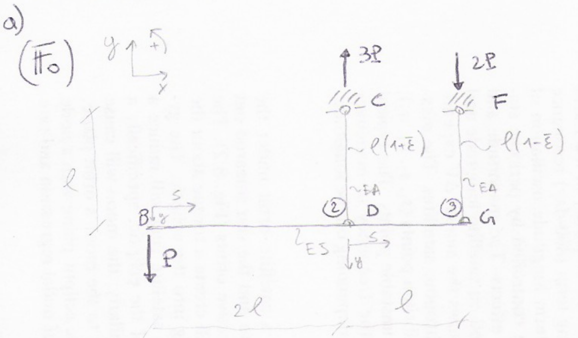
P.S. del 14/06/2004 – PARTE II

a)

(F_0)



Equilibrio della struttura:

(x) $X_{C0} + X_{F0} = 0$

(y) $Y_{C0} + Y_{F0} = P \Rightarrow \boxed{Y_{F0} = -2P}$

(F) $-\ell Y_{C0} + P\,3\ell = 0 \Rightarrow \boxed{Y_{C0} = 3P}$

Eq.ne di sconnessione in C: equilibrio rotazione asta ②

$\boxed{X_{C0} = 0} \Rightarrow \boxed{X_{F0} = 0}$

Quindi, nel sist. (F_0) l'asta ② e ③ hanno sforzo normale pari a:

$N^0_{CD} = 3P$; $N^0_{FG} = -2P$

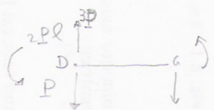
L'asta BD, $s \in [0, 2\ell]$

$N_0 = 0$

$T_0 = -P$

$\underline{M_0(s) = -Ps}$ $\to M_0(2\ell) = -2P\ell$

Tratto DG, $s \in [0, \ell]$

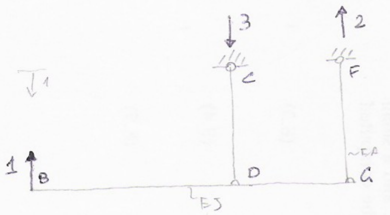


$T_0 = 2P$

$M_0 + 2P\ell + Ps - 3Ps = 0$

$\underline{M_0(s) = 2P(s - \ell)}$

(F_1)



Equilibrio della struttura

(x) $X_{C1} + X_{F1} = 0$

(y) $Y_{C1} + Y_{F1} = -1 \Rightarrow \boxed{Y_{F1} = 2}$

(F) $-Y_{C1}\ell - 1 \cdot 3\ell = 0 \Rightarrow \boxed{Y_{C1} = -3}$

Eq.ne di sconnessione in C: equilibrio rotazione asta ②

$\boxed{X_{C1} = 0} \Rightarrow \boxed{X_{F1} = 0}$

Asta ②

$N^1_{CD} = -3$

Asta ③

$N^1_{FG} = 2$

Tratto BD, $s \in [0, 2\ell]$

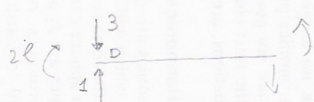
$N_1 = 0$

$T_1 = 1$

$\underline{M_1(s) = s}$ $\to M_1(2\ell) = 2\ell$

Tratto DG, $s \in [0, \ell]$

$N_1 = 0$



$T_1 = -2$

$M_1 - 2\ell - 1 \cdot s + 3s = 0$

$\underline{M_1(s) = 2(\ell - s)}$

Eq.ne di Müller-Breslau

$\eta_1 = \eta_{10} + X_1 \eta_{11}$

$\eta_1 = -\dfrac{X_1}{EA}\ell$

$$\eta_{10} = \eta_{10}^{carico} + \eta_{10}^{\bar{\varepsilon}}$$

$$\eta_{10}^{carico} = \int_0^{2\ell} \frac{(-Ps)\,s}{ES}\,ds + \int_0^{\ell} \frac{2P(s-\ell)\,2(\ell-s)}{ES}\,ds + \frac{3P\cdot(-3)}{EA}\ell + \frac{(-2P)\cdot 2}{EA}\ell = \dots\dots =$$

$$= \frac{1}{ES}\left(-\frac{8}{3}P\ell^3 - \frac{4}{3}P\ell^3\right) - \frac{9P}{EA} - \frac{4P}{EA}\Big) = -\frac{4P\ell^3}{ES} - \frac{13P\ell}{EA}$$

$$\eta_{10}^{\bar{\varepsilon}} = +\bar{\varepsilon}\,\frac{(-3)}{EA}\ell + (-\bar{\varepsilon})\frac{2}{EA}\ell = -\frac{5\bar{\varepsilon}\ell}{EA}$$

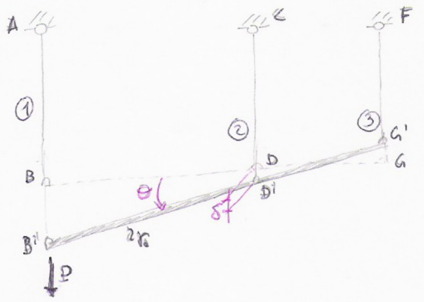
$$\eta_{11} = \int_0^{2\ell} \frac{s^2 ds}{ES} + \int_0^{\ell} \frac{4(\ell-s)^2}{ES}\,ds + \frac{(-3)^2}{EA}\ell + \frac{(2)^2}{EA}\ell = \dots\dots = \frac{8\ell^3}{3ES} + \frac{4\ell^3}{3ES} + \frac{9\ell}{EA} + \frac{4\ell}{EA} =$$

$$= \frac{4\ell^3}{ES} + \frac{13\ell}{EA}$$

Sostituisco ed ottengo:

$$\boxed{X_1 = \frac{\left(\frac{4\ell^2}{ES} + \frac{13}{EA}\right)P + \frac{5\bar{\varepsilon}}{EA}}{\frac{14}{EA} + \frac{4\ell^2}{ES}}}$$

b)

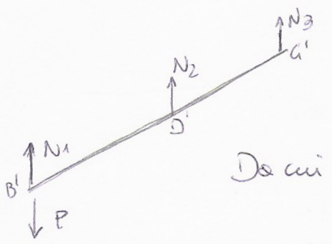


Scrivo le eq.ni costitutive per le 3 aste verticali:

$$\begin{cases} \varepsilon_1 = \frac{N_1}{EA} \\ \varepsilon_2 = \frac{N_2}{EA} + \bar{\varepsilon} \\ \varepsilon_3 = \frac{N_3}{EA} - \bar{\varepsilon} \end{cases}$$

Dalla cinematica posso scrivere che:

$$\ell\varepsilon_1 = \delta + 2\theta\ell \qquad \ell\varepsilon_2 = \delta \qquad \ell\varepsilon_3 = \delta - \theta\ell \Rightarrow \begin{cases} \varepsilon_1 = \frac{\delta}{\ell} + 2\theta \\ \varepsilon_2 = \delta/\ell \\ \varepsilon_3 = \frac{\delta}{\ell} - \theta \end{cases} \Rightarrow \begin{cases} N_1 = EA\left(\frac{\delta}{\ell} + 2\theta\right) \\ N_2 = EA\frac{\delta}{\ell} - \bar{\varepsilon} \\ N_3 = EA\left(\frac{\delta}{\ell} - \theta\right) + \bar{\varepsilon} \end{cases}$$



Ora, si deve verificare la "statica" dell'asta rigida:

- Equilibrio rotazione in D $\Rightarrow$ 1) $2EA\left(\frac{\delta}{\ell} + 2\theta\right) = EA\left(\frac{\delta}{\ell} - \theta\right) + \bar{\varepsilon} + 2P$
- Equilibrio verticale $\Rightarrow$ 2) $EA\left(\frac{\delta}{\ell} + 2\theta\right) + EA\frac{\delta}{\ell} - \bar{\varepsilon} + EA\left(\frac{\delta}{\ell} - \theta\right) + \bar{\varepsilon} = P$

Da cui $\Rightarrow$ $\boxed{\delta = \frac{3P\ell}{14EA} - \frac{\bar{\varepsilon}\ell}{14EA}}$ $\boxed{\theta = \frac{3\bar{\varepsilon}}{14EA} + \frac{5P}{14EA}}$ $\Rightarrow$ $N_1 = \frac{5}{14}\bar{\varepsilon} + \frac{13}{14}P$

che $\bar{\varepsilon} = X_1$ nel caso $ES \to \infty$!!

$N_2 = \dots$

$N_3 = \dots$