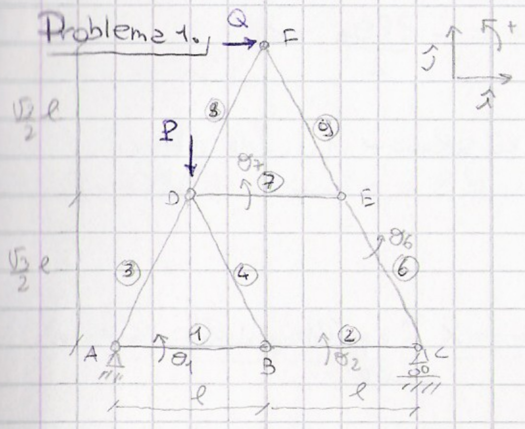
Correzione Prove Scritte SdC I, del 14/07/06

Probleme 1.



N.B.: i triangoli formati rispettivamente dalle aste (1)-(3)-(4) e (7)-(8)-(9) sono internamente connessi

$\Rightarrow$ quindi si comportano cinematicamente come un unico rigido

$$\Rightarrow \begin{cases} \theta_1 = \theta_3 = \theta_4 \\ \theta_7 = \theta_8 = \theta_9 \end{cases}$$

Condizione Vincoli

$A \longrightarrow \underline{u}_A \equiv 0$

$C \longrightarrow \underline{u}_C \cdot \hat{\jmath} = 0$

1) $\underline{u}_B = \cancel{\underline{u}_A} + \theta_1 \hat{k} \wedge \overrightarrow{AB} = \ell\theta_1 \hat{\jmath}$

$\neq \underline{u}_C = \underline{u}_B + \theta_2 \hat{k} \wedge \overrightarrow{BC} = \ell\theta_2 \hat{\jmath} + \ell\theta_1 \hat{\jmath}$
$= \ell(\theta_1 + \theta_2)\hat{\jmath}$

$\times$ le cond. del vincolo in C, $\ell(\theta_1 + \theta_2) = 0$

$\Rightarrow \boxed{\theta_2 = -\theta_1}$

$$\begin{array}{ccc} \hat{\imath} & \hat{\jmath} & k \\ 0 & 0 & -\theta_1 \\ +\frac{\ell}{2} & \frac{\sqrt{3}}{2}\ell & 0 \end{array}$$

$\underline{u}_D = \cancel{\underline{u}_A} + \theta_1 \hat{k} \wedge \overrightarrow{AD} = -\frac{\sqrt{3}}{2}\ell\theta_1 \hat{\imath} + \frac{\ell}{2}\theta_1 \hat{\jmath}$

$\underline{u}_E^{(7)} = \underline{u}_D + \theta_7 \hat{k} \wedge \overrightarrow{DE} = -\frac{\sqrt{3}}{2}\ell\theta_1 \hat{\imath} + \ell\left(\frac{\theta_1}{2} + \theta_7\right)\hat{\jmath}$

$\underline{u}_E^{(6)} = \underline{u}_C + \theta_6 \hat{k} \wedge \overrightarrow{CE} = -\frac{\sqrt{3}}{2}\ell\theta_6 \hat{\imath} - \frac{\ell}{2}\theta_6 \hat{\jmath}$

$$\underline{u}_E^{(7)} = \underline{u}_E^{(6)} \Rightarrow \begin{cases} -\frac{\sqrt{3}}{2}\ell\theta_1 = -\frac{\sqrt{3}}{2}\ell\theta_6 \\ \ell\left(\frac{\theta_1}{2} + \theta_7\right) = -\frac{\ell}{2}\theta_6 \end{cases} \Rightarrow \begin{cases} \boxed{\theta_6 = \theta_1} \\ \boxed{\theta_7 = -\theta_1} \end{cases}$$

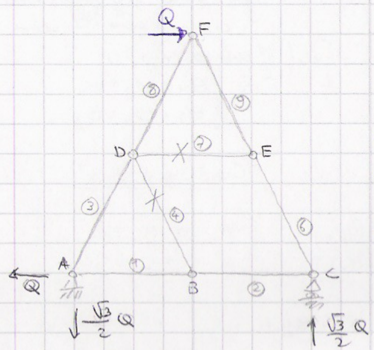
$\underline{u}_F = \underline{u}_D + \theta_7 \hat{k} \wedge \overrightarrow{DF} = -\sqrt{3}/2\, \ell\theta_1 \hat{\imath} + \ell/2\, \theta_1 \hat{\jmath} + \sqrt{3}/2\, \ell\theta_1 \hat{\imath} - \ell/2\, \theta_1 \hat{\jmath}$
$= \underline{0}!$

2) $L_V^{(e)} = 0 = L_V^{(i)}$

$L_V^{(e)} = Q \cdot \underbrace{u_F}_{0} + (-P) \cdot \frac{\ell\theta_1}{2} = -\frac{P\ell\theta_1}{2} = 0$

$\Rightarrow$ per $\theta_1 \neq 0$, la soluzione prevede $\begin{cases} P = 0 \\ \forall Q \end{cases}$

3)



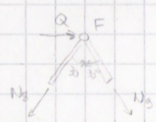
$\times$ l'equilibrio della struttura risulta:

(X): $X_A = -Q$

(A) $-Q\sqrt{3}\ell + Y_C 2\ell = 0 \Rightarrow Y_C = \frac{\sqrt{3}}{2}Q$

(Y) $Y_A = -\frac{\sqrt{3}}{2}Q$

Equilibrio nodo (F)

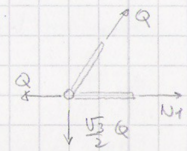


$\begin{cases} -\frac{N_8}{2} + \frac{N_9}{2} + Q = 0 & \rightarrow \begin{cases} N_9 = -Q \\ N_8 = Q \end{cases} \\ -\frac{\sqrt{3}}{2}N_8 - \frac{\sqrt{3}}{2}N_9 = 0 & \rightarrow N_8 = -N_9 \end{cases}$

Dall'equilibrio nodo E si vede $\Rightarrow N_7 = 0$; $N_6 = N_9 = -Q$

Analogamente, se N_7 è nullo, dall'eq. nodo D $\Rightarrow N_4 = 0$; $N_3 = N_8 = Q$

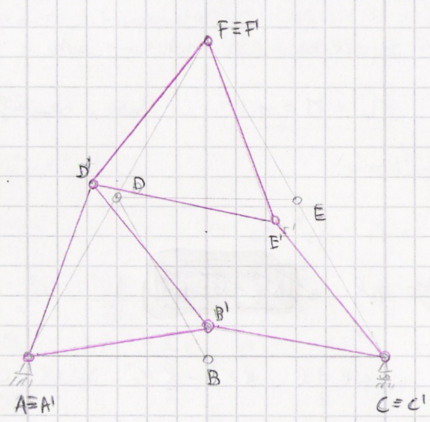
Equilibrio nodo (A)



$N_1 - Q + \frac{Q}{2} = 0 \Rightarrow N_1 = \frac{Q}{2}$

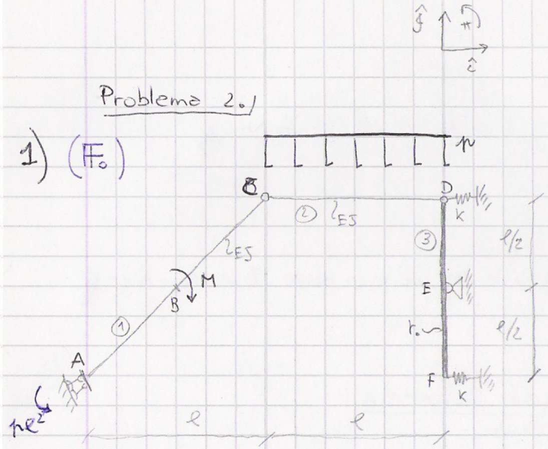
Dato che (4) è scarica, $\times$ l'equilibrio nodo B $\Rightarrow N_2 = N_1 = \frac{Q}{2}$

Disegno Struttura Deformata:



Problema 2.1

1) (F_0)



Reazioni Vincolari = 6

$N_A^\circ, M_A^\circ, X_D^\circ, X_E^\circ, X_F^\circ, Y_E^\circ$

Eq. ni:
- 3 della statica
- 1 eq. ne aux. (eq. rotazione asta 1 intorno a C)
- TRUCCHETTO!

Eq. ne aux: equilibrio alla rotazione asta 1 intorno a C

$$\boxed{M_A^\circ = M = p\ell^2}$$ (Posto x semplificare la solut.)

Eq. ne di equilibrio:

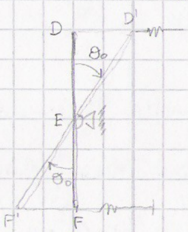
(x) : $\frac{N_A^\circ}{\sqrt{2}} + X_F^\circ + X_E^\circ + X_D^\circ = 0$

(y) $\frac{N_A^\circ}{\sqrt{2}} + Y_E^\circ = p\ell$

(A) $\underset{p\ell^2}{p\ell^2 - M} - p\ell \cdot \frac{3}{2}\ell - \ell X_D^\circ - \frac{\ell}{2} X_E^\circ + Y_E^\circ 2\ell = 0$

$$\frac{3}{2} p\ell + X_D^\circ + \frac{X_E^\circ}{2} - 2Y_E^\circ = 0$$

TRUCCHETTO : considero all'asta 3 uno spost. infinitesimo di rotazione θ_0, ed esprimo le reaz. vincolari in funzione di tale parametro



$$\begin{cases} X_D^\circ = -K \frac{\ell}{2} \theta_0 \\ X_F^\circ = K \frac{\ell}{2} \theta_0 \end{cases}$$

E poi, per l'equilibrio, impongo l'equilibrio alla rotazione intorno a D dell'asta (Eq. ne di Scorrimento)

$$X_E^\circ \frac{\ell}{2} + X_F^\circ \ell = 0$$

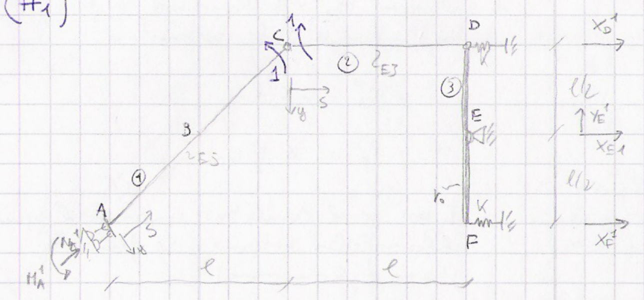
Ora ho aggiunto 1 incognita $\Rightarrow \begin{cases} \text{INCOGNITE} = 7 \\ \text{EQUAZIONI} = 7 \end{cases}$

$$\begin{cases} M_A^\circ = p\ell^2 \\ \frac{N_A^\circ}{\sqrt{2}} + X_F^\circ + X_E^\circ + X_D^\circ = 0 \\ \frac{N_A^\circ}{\sqrt{2}} + Y_E^\circ = p\ell \\ \frac{3}{2} p\ell + X_D^\circ + \frac{X_E^\circ}{2} - 2Y_E^\circ = 0 \\ X_D^\circ = -K\frac{\ell}{2}\theta_0 \;;\; X_F^\circ = K\frac{\ell}{2}\theta_0 \Rightarrow X_D^\circ = -X_F^\circ \\ X_E^\circ \frac{\ell}{2} + X_F^\circ \ell = 0 \end{cases}$$

La cui soluzione mi dà :

$$\boxed{\begin{matrix} N_A^\circ = \frac{\sqrt{2}}{2} p\ell \;; & X_E^\circ = -\frac{p\ell}{2} \\ X_F = \frac{p\ell}{4} \;; & X_D^\circ = -\frac{p\ell}{4} \\ Y_E = \frac{p\ell}{2} \;; & \{M_A^\circ = p\ell^2\} \end{matrix}}$$

(F1)



Analogamente al sistema F0, anche qui le incognite risulteranno essere 7 (compreso θ_1), e le equazioni portanti 7 (con l'utilizzo sempre del TRUCCHETTO!)

- Eq.ne aux. nodo C, per asta ①:

$$1 + M_A^1 = 0 \Rightarrow \boxed{M_A^1 = -1}$$

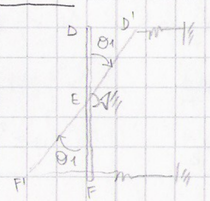
- Eq.ni della statica per la struttura:

(x): $\frac{N_A^1}{\sqrt{2}} + X_F^1 + X_E^1 + X_D^1 = 0$

(y): $\frac{N_A^1}{\sqrt{2}} - Y_E^1 = 0$

(A): $-1 + 1 - 1 + 2\ell Y_E^1 - \frac{\ell}{2} X_E^1 - \ell X_D^1 = 0$

TRUCCHETTO!:



$$\begin{cases} X_D^1 = -K \frac{\ell}{2} \theta_1 \\ X_F^1 = K \frac{\ell}{2} \theta_1 \end{cases}$$

X l'equilibrio alla rotazione asta ③ intorno ad D:

$$X_E^1 \frac{\ell}{2} + X_F^1 \ell = 0$$

Quindi il sistema risolutivo diventa:

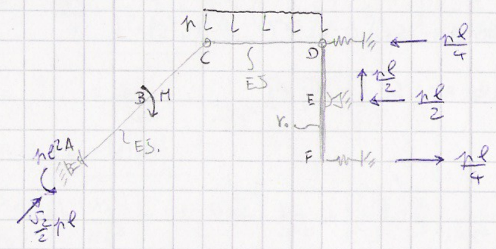
$$\begin{cases} \frac{N_A^1}{\sqrt{2}} + X_F^1 + X_E^1 + X_D^1 = 0 \\ \frac{N_A^1}{\sqrt{2}} + Y_E^1 = 0 \\ 2Y_E^1 - \frac{X_E^1}{2} - X_D^1 = \frac{1}{\ell} \\ X_D^1 = -X_F^1 \\ \frac{X_E^1}{2} + X_F^1 = 0 \end{cases}$$

La cui soluzione è:

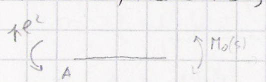
$$\boxed{N_A^1 = -\frac{\sqrt{2}}{\ell};\ X_E^1 = Y_E^1 = \frac{1}{\ell} \quad X_F^1 = -\frac{1}{2\ell};\ X_D^1 = \frac{1}{2\ell};\ M_A^1 = -1}$$

Ora bisogna calcolare le CdS nei due sistemi:

Sistema (F0)



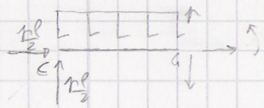
Asta AC, tratto AB; $s \in [0, \frac{\sqrt{2}}{2}\ell]$



$$\boxed{M_0(s) = -p\ell^2}$$

Asta AC, tratto BC; $s \in [0, \frac{\sqrt{2}}{2}\ell] \Rightarrow M_0(s) = 0$

Asta CD, $s \in [0, \ell]$



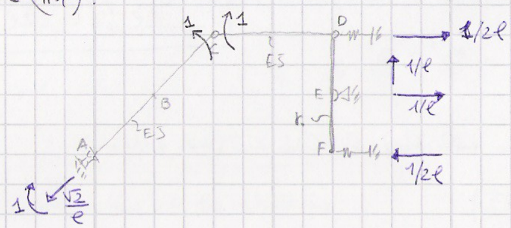
$N_0(s) = -\frac{p\ell}{2}$
$T_0(s) = \frac{p\ell}{2}$

$M_0(s) = ps \cdot \frac{s}{2} - \frac{p\ell s}{2} = 0$

$\boxed{M_0(s) = \frac{ps}{2}(\ell - s)}$

L'asta ③ è rigida ⇒ niente contributo al momento

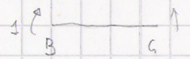
Sistema (Π_1):



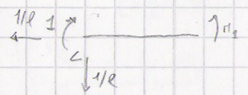
Asta ①, tratto AB; $s \in [0, \frac{\sqrt{2}}{2}\ell] \Rightarrow \boxed{M_1(s) = 1}$

" ", tratto BC; $s \in [0, \frac{\sqrt{2}}{2}\ell]$

$\boxed{M_1(s) = 1}$



Asta②, $s \in [0, \ell]$:



$M_1(s) - 1 + \frac{1}{\ell}s = 0$

$\boxed{M_1(s) = 1 - \frac{s}{\ell}}$

Eq.ne di Müller-Breslau:

$$\eta_1 = \eta_{10} + X_1 \eta_{11} + \eta_{1\ell}$$

$$\eta_1 = -\frac{X_1}{K_0}$$

$$\eta_{10} = \int_\ell \frac{M_0 M_1}{EJ} ds = \int_0^{\frac{\sqrt{2}}{2}\ell} (-p\ell^2)\frac{1}{EJ} ds + \int_0^\ell \frac{ps}{2}(\ell - s)\left(1 - \frac{s}{\ell}\right)\frac{1}{EJ} ds =$$

$$= \dots = (1 - 12\sqrt{2})\frac{p\ell^3}{24EJ} = (1 - 12\sqrt{2})\frac{p}{24K}$$

$$\eta_{11} = \int_\ell \frac{M_1^2}{EJ} ds = 2\int_0^{\frac{\sqrt{2}}{2}\ell} \frac{1}{EJ} ds + \int_0^\ell \left(1 - \frac{s}{\ell}\right)^2 \frac{1}{EJ} ds = \dots = \left(\sqrt{2} + \frac{1}{3}\right)\frac{\ell}{EJ} =$$

$$= \left(\sqrt{2} + \frac{1}{3}\right)\frac{1}{K\ell^2}$$

$$\eta_{1\ell} = \sum_v \frac{R_0 R_1}{K_v} + X_1 \sum_v \frac{R_1^2}{K_v} =$$

$$= -\frac{p\ell}{4} \cdot \frac{1}{2\ell}\frac{1}{K} + \frac{p\ell}{4}\left(-\frac{1}{2\ell}\right)\frac{1}{K} + X_1\left[\left(-\frac{1}{2\ell}\right)^2\frac{1}{K} + \left(\frac{1}{2\ell}\right)^2\frac{1}{K}\right] =$$

$$= \dots = -\frac{p}{4K} + \frac{1}{2K\ell^2}X_1$$

Sostituisco

$$-\frac{X_1}{K_0} = -\frac{X_1}{K\ell^2} = (1 - 12\sqrt{2})\frac{p}{24K} + \left[\left(\sqrt{2} + \frac{1}{3}\right)\frac{1}{K\ell^2}\right]X_1 - \frac{p}{4K} + \frac{1}{2K\ell^2}X_1$$

$$\dots \Rightarrow \boxed{X_1 = \left(\frac{5 + 12\sqrt{2}}{11 + 6\sqrt{2}}\right)\frac{p\ell^2}{4}} \simeq 0.2819\, p\ell^2$$

2) Asta ① $\overset{\circ}{AC}$: $N_{AC} = N^0_{AC} + X_1 N^1_{AC} = -\frac{\sqrt{2}}{2} p\ell + X_1 \frac{\sqrt{2}}{\ell}$

$\Rightarrow N_{AC} \simeq -0.71\, p\ell + 0.40\, p\ell \simeq -0.31\, p\ell$

$T_{AC} \equiv 0$

Tratto AB

$M_{AB} = -p\ell^2 + X_1 =$

$\simeq -0{,}72\, p\ell^2$

Tratto BC

$M_{BC} = X_1 \cong 0{,}28\, p\ell^2$

Asta ②

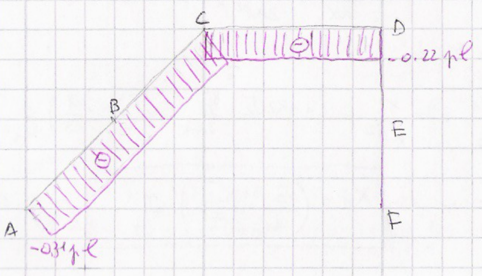
$N_{CD} = -\frac{p\ell}{2} + X_1 \frac{1}{\ell} = -0.5\, p\ell + 0.28\, p\ell \simeq -0.22\, p\ell$

$T_{CD} = \frac{p\ell}{2} + X_1 \left(-\frac{1}{\ell}\right) = 0.5\, p\ell - 0.28\, p\ell \simeq 0.22\, p\ell$

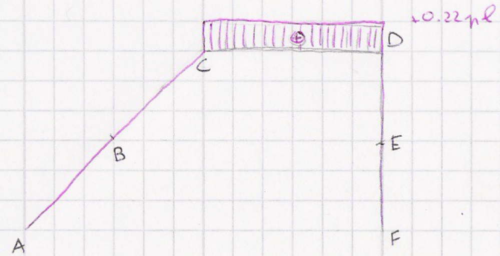
$M_{CD} = \frac{ps}{2}(\ell - s) + X_1\left(1 - \frac{s}{\ell}\right) = \dots \cong -0.5\, ps^2 + 0.22\, p\ell s + 0.28\, p\ell^2$

Asta ③ $\longrightarrow$ RIGIDA

N



T



M